

D.A.V PUBLIC SCHOOLS, JHARKHAND ZONE G

Pre-Board 2023-2024

Subject :Mathematics(041)

Class :12

Time allowed:3 hours

Maximum Marks:80

General Instructions:

1. This Question paper contains-five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 2 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION-A

(Multiple Choice Question) :Each question carries 1 mark

1. If A is a square Matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to
(a) A (b) $I-A$ (c) I (d) $3A$
2. If $A(\text{adj}A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then the value of $|A| + |\text{adj} A|$ is equal to
(a) 8 (b) 12 (c) 24 (d) 16
3. If the area of the triangle with vertices $(-2,0)$, $(0,4)$ and $(0,K)$ is 4. Sq units, then the value of K will be
(a) $(0,8)$ (b) ± 8 (c) 0 (d) 8
4. If $f(x) = \begin{cases} kx + 1 & \text{if } x \leq \pi \\ \cos x & \text{if } x > \pi \end{cases}$ is continuous at $x=\pi$, then the value of K is
(a) $\frac{2}{\pi}$ (b) $-\frac{2}{\pi}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{2}$
5. If the direction cosines of the line are $K/3, K/3, K/3$ then value of K is
(a) $K > 0$ (b) $0 < K < 1$ (c) ± 3 (d) $\pm \sqrt{3}$
6. The sum of order and degree of the differential Equation
$$\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{1}{3}} = \frac{d^2y}{dx^2}$$
 is
(a) 2 (b) 5 (c) $\frac{1}{3}$ (d) not defined

7. The objective function of linear programming problem is

- (a) A constraint
- (b) function to be optimised
- (c) A relation between the variables
- (d) None of these

8. The value of the expression $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2$ is

- (a) $|\vec{a}|^2 |\vec{b}|^2$
- (b) $|\vec{a}|^2 |\vec{b}|$
- (c) $|\vec{a}| |\vec{b}|^2$
- (d) $|\vec{a}| |\vec{b}|$

9. If $f(x) = \int_0^x t \sin t dt$, then $f'(x)$ is

- (a) $\cos x + x \sin x$
- (b) $x \cos x$
- (c) $x \sin x$
- (d) $\sin x + x \cos x$

10. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then

- (a) $|3A| = 9|A|$
- (b) $3|A| = 3|A|$
- (c) $|3A| = 4$
- (d) $|3A| = 27|A|$

11. The value of objective function is maximum under the linear constraints:

- (a) at the centre of feasible region
- (b) at origin
- (c) at any vertex of feasible region
- (d) vertex which is at maximum distance from origin

12. If $\vec{a} = 4\hat{i} + 6\hat{j}$ and $\vec{b} = 3\hat{j} + 4\hat{k}$ the vector form of the component of $\vec{a}$ along $\vec{b}$ is

- (a) $\frac{18}{5}(3\hat{i} + 4\hat{k})$
- (b) $\frac{18}{25}(3\hat{j} + 4\hat{k})$
- (c) $\frac{18}{5}(3\hat{i} + 4\hat{k})$
- (d) $\frac{18}{25}(4\hat{i} + 6\hat{j})$

13. Assume X, Y, Z, W and P are matrices of order $2 \times n$, $3 \times K$, $2 \times p$, $n \times 3$ and $p \times k$ so that $PY + WY$ will be defined are

- (a) $K=3, p=n$
- (b) K is arbitrary, $p=2$
- (c) P is arbitrary, $K=3$
- (d) $K=2, p=3$

14. A problem in mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the exactly one of them solves the problem

- (a) $\frac{3}{4}$
- (b) $\frac{2}{3}$
- (c) $\frac{1}{2}$
- (d) $\frac{11}{24}$

15. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^{-x} + e^y = C$ (b) $e^x + e^y = C$ (c) $e^x + e^{-y} = C$ (d) $e^{-x} + e^{-y} = C$

16. For any two vectors $\vec{a}$ and $\vec{b}$, we always have

(a) $|\vec{a} \cdot \vec{b}| > |\vec{a}| |\vec{b}|$ (b) $|\vec{a} \cdot \vec{b}| \geq |\vec{a}| |\vec{b}|$ (c) $|\vec{a} \cdot \vec{b}| < |\vec{a}| |\vec{b}|$ (d) $|\vec{a} \cdot \vec{b}| \leq |\vec{a}| |\vec{b}|$

17. If $f(x) = x|x|$ then $f'(-1)$ is

(a) 2 (b) 1 (c) -2 (d) -1

18. Direction cosines of a line $6x - 2 = 3y + 1 = 2z - 4$ are

(a) $\frac{1}{\sqrt{54}}, \frac{-7}{\sqrt{54}}, \frac{2}{\sqrt{54}}$ (b) $\frac{3}{7}, \frac{2}{7}, \frac{-6}{7}$ (c) $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$ (d) $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}$

ASSERTION-BASED REASON QUESTIONS

In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct out of the following choices.

- (a) both (A) and (R) are true and (R) is the correct explanation of (A)
 (b) both (A) and (R) are true but (R) is not the correct explanation of (A)
 (c) (A) is true and (R) is false
 (d) (A) is false and (R) is true

19. Assertion (A)

The maximum value of the function $f(x) = x^5, x \in [-1, 1]$ is attained at critical point $x = 1$.

Reason (R)

The maximum value of a function can occur only at points where derivative is zero

20. Assertion (A):

The relation $f: \{1, 2, 3, 4\} \rightarrow \{x, y, z\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function

Reason (R):

The function $f: \{1, 2, 3\} \rightarrow \{x, y, z\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

SECTION -B

21. Find the domain of $\sec^{-1}(3x - 2)$?

OR

Find the value of $\tan^{-1}(1) + \cot^{-1}\left(\frac{-1}{\sqrt{3}}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

22. Find the intervals in which the function f given by $f(x) = \cos 3x, 0 < x < \frac{\pi}{2}$ is increasing or decreasing.

23. Find the difference between the greatest and least values of the function $f(x) = \sin 2x - x$ on $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$.

OR

Find the local maxima and minima values of the function f given by $f(x) = 3x^4 + 4x^3 - 12x^2 + 12$.

24. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$.

25. A man of height 2 meters walks at a uniform speed of 5 km/hr away from a lamp post which is 6 meters high. Find the rate at which the length of his shadow increases.

SECTION-C

This section comprises of short answer type questions(SA) of 3 marks each

26. Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$.

27. Find the probability distribution of number of doublets in three throws of a pair of dice.

28. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$.

OR

Evaluate $\int \frac{(x-3)e^x}{(x-1)^3} dx$.

29. Find the general solution $\frac{dy}{dx} + 2y = \sin x$.

OR

Solve the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$.

30. Solve the linear programming problem graphically

Maximise

$$Z = 1000x + 600y$$

Subject to constraints

$$x + y \leq 200; \quad x \geq 20; \quad y - 4x \geq 0; \quad x \geq 0; \quad y \geq 0.$$

OR

Solve the following linear programming problem graphically.

Minimize: $z = 60x + 80y$

Subject to constraints : $3x + 4y \geq 8; \quad 5x + 2y \geq 11; \quad x \geq 0; \quad y \geq 0.$

31. If $y = x^x$ prove that $\frac{d^2y}{dx^2} - \frac{1}{y} \left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$.

SECTION-D

This section comprises of long answer (LA) type questions of 5 marks each

32. Find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$. Using the method of integration.

33. Let N denote the set of all natural numbers and R is the relation on $N \times N$ defined by $(a,b)R(c,d) \Leftrightarrow ad(b+c)=bc(a+d)$. Check whether R is an equivalence relation.

OR

Consider $f: \mathbb{R}^+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is bijective.

34. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$ find A^{-1} . Using A^{-1} solve the system of equations.

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3$$

35. Find the cartesian and vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the lines. Also find the angle between the given lines.

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$$

and

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$

OR

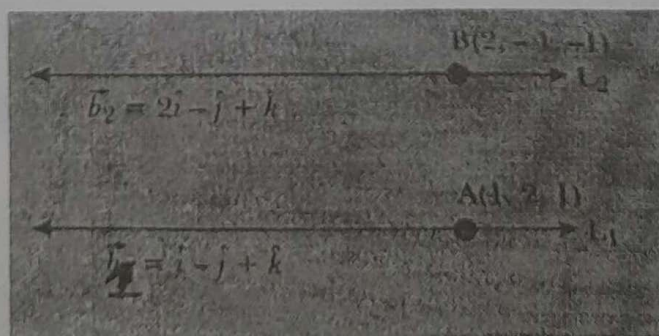
Find the co-ordinate of the foot of the perpendicular drawn from the point $A(1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. Also find the length of the perpendicular.

SECTION -E

This section comprises of 3 case-study/passage-based questions of 4 marks each. First two case study questions have three sub-parts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)

36. Read the following passage and answer the questions given below.

A teacher asked a student to draw two skew lines. The student draws two skew lines L_1 and L_2 shown below with their points through which they pass and their directions respectively.



- (i) Write the vector equation of the line L_1
OR

Write the cartesian equation of the line L_2 .

- (ii) Find the value of $\vec{b}_1 \times \vec{b}_2$.
(iii) Find the shortest distance between given two skew lines.

37. $P(x) = -6x^2 + 120x + 25000$ (in) is the total profit function of a company, where x denotes the production of the company.



Based on the above information, answer the following questions.

- (i) Find the profit of the company, when the production is 3 units.
(ii) Find $P'(5)$.
(iii) Find the interval in which the profit is strictly increasing.

Or

Find the production, when the profit is maximum.

38. In an office three employees Vinay, Sonia and Iqbal process incoming copies of a certain form. Vinay process 50% of the forms. Sonia processes 20% and Iqbal the remaining 30% of the forms. Vinay has an error rate of 0.06, Sonia has an error rate of 0.04 and Iqbal has an error rate of 0.03.



Based on the above information answer the following questions.

- (i) The total probability of committing an error in processing the form.
(ii) The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is not processed by Vinay.